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APPLICATION OF LANCZOS VECTORS TO CONTROL DESIGN OF FLEXIBLE STRUCTURES — PART 2

by
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ABSTRACT

This report covers the period of the grant from January 1991 until its expiration in June 1992. Together with an Interim Report [Ref. 9], it summarizes the research conducted under NASA Grant NAG9-357 on the topic "Application of Lanczos Vectors to Control Design of Flexible Structures." The research concerns various ways to obtain reduced-order mathematical models of complex structures for use in dynamics analysis and in the design of control systems for these structures. This report summarizes research described in the following reports and papers that were written during the second half of the grant period.

Papers Published in Refereed Journals

- Su, Tzu-Jeng and Craig, Roy R., Jr., "Model Reduction and Control of Flexible Structures Using Krylov Vectors," Ref. [1].
- Su, Tzu-Jeng and Craig, Roy R., Jr., "Krylov Model Reduction for Undamped Structural Dynamics Systems," Ref. [2].
- Su, Tzu-Jeng and Craig, Roy R., Jr., "Control Design Based on a Linear State Function Observer," Ref. [3].
- Su, Tzu-Jeng and Craig, Roy R., Jr., "An Unsymmetric Lanczos Algorithm For Damped Structural Dynamics Systems," Ref. [4].

Book Chapters

- Su, Tzu-Jeng and Craig, Roy R., Jr., 'Krylov Vector Methods for Model Reduction and Control of Flexible Structures," Ref. [5].

Papers Presented at Technical Meetings

- Su, Tzu-Jeng and Craig, Roy R., Jr., "Structural Control Design Based on Reduced-Order Observer," Ref. [6].
- Su, Tzu-Jeng and Craig, Roy R., Jr., "An Unsymmetric Lanczos Algorithm For Damped Structural Dynamics Systems," Ref. [7].
- Craig, Roy R., Jr., Su, Tzu-Jeng, and Kim, Hyoung M., "Some Experiences with Krylov Vectors and Lanczos Vectors," Ref. [8].

Bound Reports

- Craig, Roy R., Jr. and Su, Tzu-Jeng, "Application of Lanczos Vectors to Control Design of Flexible Structures," Ref. [9].
- Su, Tzu-Jeng and Craig, Roy R., Jr., "Unsymmetric Lanczos Model Reduction and Linear State Function Observer for Flexible Structures," Ref. [10].

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Chapter 1

INTRODUCTION

Together with Ref. 9, this report summarizes the research accomplished over a three-year period on the topic of the application of Krylov vectors and Lanczos vectors to the control of flexible structures. The goals stated for research under the grant are:

1. To develop a theory of reduced-order modeling of general linear systems based on the use of Lanczos vectors, and to apply the theory to the modeling of flexible structures.

2. To address numerical issues that arise in the application of Lanczos vectors to reduced-order modeling, for example, sensitivity to choice of starting vectors, loss of orthogonality, etc.

3. To develop control system design techniques employing Lanczos modeling of the controlled and residual systems, considering relevant issues such as stability of the closed-loop system, spillover, robustness, and computational requirements.

4. To apply Lanczos-based control system design to several typical problems, for example, optimal co-located velocity feedback, dynamic output feedback, optimal control of finite-time slewing of a beam, etc.

Algorithms for reduced-order modeling of general linear systems and of damped and undamped structures (Goal 1) are described in Refs. [1,2,4,7,10,11, 14, and 15]. Steps that may be taken to address several numerical issues that arise in the application of Lanczos vectors to reduced-order modeling (Goal 2) are described in Refs. [7,10–12]. References [1,3,6,10,11,13–15,20] address the topic of control system design techniques (Goals 3 and 4). Summary papers and reports covering research performed under the present grant appear in Refs. [5,8,9,16].

Chapter 2 contains abstracts of the major research publications and presentations not previously summarized in the Interim Report [Ref. 9].

Chapter 2

ABSTRACTS OF TECHNICAL PAPERS

In Ref. [9], which covers research accomplished during the first eighteen months of this grant, abstracts were presented on the following topics:

- Controller Reduction by Preserving Impulse Response Energy
- Substructuring Decomposition and Controller Synthesis
- A Review of Model Reduction Methods for Structural Control Design
- Recent Literature on Structural Modeling, Identification, and Analysis

Abstracts of publications describing research conducted during the last eighteen months of the grant period are presented in this chapter.

2.1 Krylov Model Reduction (Refs. 1, 2, 5, 10)

Some studies have shown that Krylov/Lanczos-based reduced-order models provide an alternative to normal-mode (eigenvector) reduced-order models in application to structural control problems. The formulation based on Krylov vectors can eliminate control spillover and observation spillover while leaving only the dynamic spillover terms to be considered.

An undamped structural dynamics system can be described by the input-output equations

$$\begin{aligned} M\ddot{x} + Kx &= Pu \qquad x \in R^n, \;\; u \in R^l \\ y &= Vx + W\dot{x} \qquad y \in R^m \end{aligned} \tag{1}$$

Model reduction of structural dynamics systems is usually based on the Rayleigh-Ritz method of selecting an $n \times r$ transformation matrix L such that

$$x = L\bar{x} \tag{2}$$

where $\bar{x} \in R^r$ $(r < n)$ is the reduced-order vector of (physical or generalized) coordinates. Then, the reduced system equation is given by

$$\begin{aligned} \overline{M}\ddot{\bar{x}} + \overline{K}\bar{x} &= \overline{P}u \\ y &= \overline{V}\bar{x} + \overline{W}\dot{\bar{x}} \end{aligned} \tag{3}$$

where $\overline{M} = L^T M L$, $\overline{K} = L^T K L$, $\overline{P} = L^T P$, $\overline{V} = VL$, and $\overline{W} = WL$. The projection matrix L can be chosen arbitrarily. However, by choosing L to be formed by a particular set of Krylov vectors, it can be shown that the resulting reduced-order model matches a set of parameters called *low-frequency moments*.

For a general linear system

$$\begin{aligned} \dot{z} &= Az + Bu \qquad z \in R^n, \; u \in R^l \\ y &= Cz \qquad\qquad y \in R^m \end{aligned} \tag{4}$$

the low-frequency moments are defined by $CA^{-i}B$, $i = 1, 2, \ldots$, which are the coefficient matrices in the Taylor series expansion of the system transfer function. Applying the Fourier transform to Eq. (1a) yields the frequency response solution $X(\omega) = (K - \omega^2 M)^{-1} PU(\omega)$, with $X(\omega)$ and $U(\omega)$ the Fourier transforms of x and u. If the system is assumed to have no rigid-body motion, then a Taylor expansion of the frequency response around $\omega = 0$ is possible. Thus,

$$X(\omega) = (I - \omega^2 K^{-1} M)^{-1} K^{-1} PU(\omega) = \sum_{i=0}^{\infty} \omega^{2i} (K^{-1} M)^i K^{-1} PU(\omega) \qquad (5)$$

Combining Eq. (1b) and Eq. (5), the system output frequency response can be expressed as

$$Y(\omega) = \sum_{i=0}^{\infty} [V(K^{-1} M)^i K^{-1} P + j\omega W(K^{-1} M)^i K^{-1} P] \omega^{2i} U(\omega) \qquad (6)$$

In these expressions, $V(K^{-1}M)^i K^{-1}P$ and $W(K^{-1}M)^i K^{-1}P$ play roles similar to that of low-frequency moments in the first-order state-space formulation. To obtain the reduced-order model of Eqs. (3) let

$$span\ \{L\} = span\ \{L_P\ L_V\ L_W\} \qquad (7)$$

where

$$\begin{aligned} L_P &= \left[\begin{array}{cccc} K^{-1}P & (K^{-1}M)K^{-1}P & \cdots & (K^{-1}M)^p K^{-1}P \end{array} \right] \\ L_V &= \left[\begin{array}{cccc} K^{-1}V^T & (K^{-1}M)K^{-1}V^T & \cdots & (K^{-1}M)^q K^{-1}V^T \end{array} \right] \\ L_W &= \left[\begin{array}{cccc} K^{-1}W^T & (K^{-1}M)K^{-1}W^T & \cdots & (K^{-1}M)^s K^{-1}W^T \end{array} \right] \end{aligned} \qquad (8)$$

for $p,\ q,\ s \geq 0$. Then the reduced system matches the low frequency moments $V(K^{-1}M)^i K^{-1}P$ for $i = 0,\ 1,\ \ldots,\ p + q + 1$ and $W(K^{-1}M)^i K^{-1}P$, for $i = 0,\ 1,\ 2,\ \ldots,\ p + s + 1$.

The L_P matrix above is the *generalized controllability matrix*, and the L_V and L_W matrices are the *generalized observability matrices* of the dynamic system described by Eq. (1). The vectors contained in L_P are *Krylov vectors* that are generated in block form by

$$Q_1 = K^{-1}P$$
$$Q_{i+1} = K^{-1}MQ_i$$

The first vector block, $K^{-1}P$, is the system's static deflection due to the force distribution P. The vector block Q_{i+1} can be interpreted as the static deflection produced by the inertia force associated with the Q_i. If only the dynamic response simulation is of interest, then $L = L_P$ would be chosen. In this case, the reduced model matches $p + 1$ low-frequency moments. As to the vectors in L_V and L_W, a physical interpretation such as the "static deflection due to sensor distribution" may be inadequate. However, from an input-output point of view, L_V, L_W, and L_P are equally important as far as parameter-matching of the reduced-order model is concerned.

Based on Eq. (7), the following algorithm may be used to generate a Krylov basis that produces a reduced-order model with the stated parameter-matching property.

Krylov/Lanczos Algorithm

(1) *Starting block of vectors*:

(a) $Q_0 = 0$

(b) $R_0 = K^{-1}\tilde{P}$, $\quad \tilde{P} =$ *linearly-independent portion of* $[P\ V^T\ W^T]$

(c) $R_0^T K R_0 = U_0 \Sigma_0 U_0^T$ $\qquad$ (*singular-value decomposition*)

(d) $Q_1 = R_0 U_0 \Sigma_0^{-\frac{1}{2}}$ *(normalization)*

(2) *For* $j = 1, 2, \ldots, k-1$, *repeat*:

(e) $\overline{R}_j = K^{-1} M Q_j$

(f) $R_j = \overline{R}_j - Q_j A_j - Q_{j-1} B_j$ *(orthogonalization)*

$A_j = Q_j^T K \overline{R}_j, \quad B_j = U_{j-1} \Sigma_{j-1}^{\frac{1}{2}}$

(g) $R_j^T K R_j = U_j \Sigma_j U_j^T$ *(singular-value decomposition)*

(h) $Q_{j+1} = R_j B_{j+1}^{-T} = R_j U_j \Sigma_j^{-\frac{1}{2}}$ *(normalization)*

(3) *Form the k-block projection matrix* $L = \left[\begin{array}{cccc} Q_1 & Q_2 & \cdots & Q_k \end{array} \right]$.

This algorithm is a Krylov algorithm, because the L matrix is generated by a Krylov recurrence formula (Step e). It is a Lanczos algorithm because the orthogonalization scheme is a three-term recursion scheme (Step f).

If the projection matrix L generated by the above algorithm is employed to perform model reduction, then the reduced-order model matches the low-frequency moments $V(K^{-1}M)^i K^{-1} P$ and $W(K^{-1}M)^i K^{-1} P$, for $i = 0, 1, 2, \ldots, 2k-1$. It can also be shown that the reduced-order model approximates the lower natural frequencies of the full-order model.

One interesting feature of the transformed system equation in Krylov coordinates is that it has a special form. Because of the special choice of starting vectors, K-orthogonalization, and three-term recurrence, the transformed system equation has a mass matrix in block-tridiagonal form, a stiffness matrix equal to the identity matrix, and force distribution and measurement distribution matrices with nonzero elements only in the first block. The transformed

system equation has the form

$$\begin{bmatrix} \times & \times & & & & & \\ \times & \times & \times & & & & \\ & \times & \times & \cdot & & & \\ & & \cdot & \cdot & \cdot & & \\ & & & \cdot & \cdot & \cdot & \\ & & & & \cdot & \cdot & \times \\ & & & & & \times & \times \end{bmatrix} \ddot{\bar{x}} + \bar{x} = \begin{Bmatrix} \times \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{Bmatrix} u \tag{9}$$

$$y = [\ \times \ \ 0 \ \ 0 \ \ \cdots \ \ 0\]\bar{x} + [\ \times \ \ 0 \ \ 0 \ \ \cdots \ \ 0\]\dot{\bar{x}}$$

where $\times$ denotes the location of nonzero elements. This special form reflects the structure of a tandem system (Fig. 1), in which only subsystem S_1 is directly

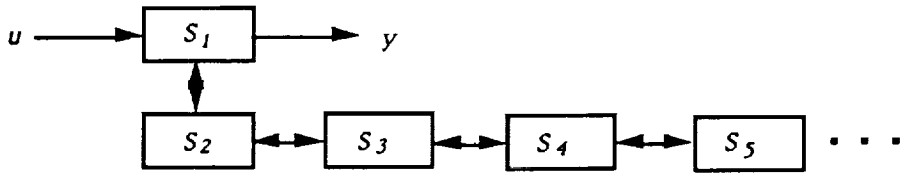


Figure 1. The Structure of a Tandem System.

controlled and measured while the remaining subsystems, S_i, $i = 2, 3, \ldots$, are excited through chained dynamic coupling. In control applications, this tandem structure of the dynamic equation eliminates the control spillover and the observation spillover, but there is still dynamic spillover. For dynamic response calculations, the block-tridiagonal form can lead to an efficient time-step solution and can save storage.

The previous model-reduction strategy can be extended to damped structural dynamics systems, which are described by the linear input-output equations

$$\begin{aligned} M\ddot{x} + D\dot{x} + Kx &= Pu \\ y &= Vx + W\dot{x} \end{aligned} \tag{10}$$

To arrive at an algorithm for constructing a reduced-order model that matches low-frequency moments, it is easier to start from the first-order formulation. The first-order differential equation equivalent to Eq. (10) can be expressed as

$$\begin{aligned} &\begin{bmatrix} D & M \\ M & 0 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} = \begin{Bmatrix} P \\ 0 \end{Bmatrix} u \\ &y = [V \quad W] \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} \end{aligned} \tag{11}$$

or

$$\begin{aligned} \hat{M}\dot{z} + \hat{K}z &= \hat{P}u \\ y &= \hat{V}z \end{aligned} \tag{12}$$

with

$$\hat{M} = \begin{bmatrix} D & M \\ M & 0 \end{bmatrix}, \quad \hat{K} = \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix}, \quad \hat{P} = \begin{Bmatrix} P \\ 0 \end{Bmatrix}, \quad \hat{V} = \begin{bmatrix} V & W \end{bmatrix} \tag{13}$$

This leads to the following recurrence formula for the Krylov blocks:

$$\begin{bmatrix} Q_{j+1}^d \\ Q_{j+1}^v \end{bmatrix} = \begin{bmatrix} -K^{-1}D & -K^{-1}M \\ I & 0 \end{bmatrix} \begin{bmatrix} Q_j^d \\ Q_j^v \end{bmatrix} \tag{14}$$

Superscripts d and v denote displacement and velocity portions of the vector, respectively. The matrix containing the generated vector sequence is called a Krylov matrix. It has the form

$$\begin{bmatrix} Q_1^d & Q_2^d & Q_3^d & \cdots \\ Q_1^v & Q_1^d & Q_2^d & \cdots \end{bmatrix}$$

Krylov subspaces that are generated by Eq. (14) and that have the above form produce a projection subspace L that has the desired moment-matching property.

Let

$$L_{\hat{P}} = \begin{bmatrix} Q_1^d & Q_2^d & Q_3^d & \cdots & Q_p^d \\ 0 & Q_1^d & Q_2^d & \cdots & Q_{p-1}^d \end{bmatrix} \tag{15a}$$

be the sequence of vectors generated by Eq. (14) with $\hat{K}^{-1}\hat{P}$ the starting block of vectors, i.e., $Q_1^d = K^{-1}P$, $Q_1^v = 0$, and let

$$L_{\hat{V}} = \begin{bmatrix} P_1^d & P_2^d & P_3^d & \cdots & P_q^d \\ P_1^v & P_1^d & P_2^d & \cdots & P_{q-1}^d \end{bmatrix} \tag{15b}$$

be the subspace of vectors generated by Eq. (14) with $\hat{K}^{-1}\hat{V}$ the starting block of vectors, i.e., $P_1^d = K^{-1}V^T$, $P_1^v = -M^{-1}W^T$. If the projection matrix L is chosen such that

$$span\ \{L\} = span\ \left\{ \begin{array}{cccccccc} Q_1^d & \cdots & Q_p^d & P_1^d & \cdots & P_q^d & P_1^v \end{array} \right\} \tag{16}$$

then the reduced-order model of the damped structural dynamics system matches the system parameters $\hat{V}(\hat{K}^{-1}\hat{M})^i\hat{K}^{-1}\hat{P}$, for $i = 0,\ 1,\ \ldots,\ p+q-1$.

A 48-DOF plane truss structure is used in Reference [1] to demonstrate the Krylov model reduction method. It is shown that the Krylov-based reduced-models can approximate the system's impulse response better than the normal-mode reduced-models. The same structure is also used to illustrate the efficacy of Krylov vector method in the application to flexible structure control problems.

2.2 Unsymmetric Lanczos Algorithms for Damped Structural Dynamics Systems (Refs. 4, 7, 10, 12)

The Krylov model reduction method described in the previous section has a restriction: the damping matrix has to be symmetric. Although most passive damping mechanisms yield a symmetric damping matrix, there are cases when the damping matrix is unsymmetric. For structures, unsymmetric damping may arise from either active feedback control or Coriolis forces. To deal with general, nonsymmetric damping, the usual approach is to write the system's dynamic equation in first-order state-space form. Then, an unsymmetric Lanczos algorithm is used to create a basis for model reduction of the first-order differential equations. All the existing unsymmetric Lanczos algorithms are two-sided [12]. In this section, a one-sided, three-term, block-Lanczos iteration scheme will be presented. This iteration scheme, together with a special choice of starting block of vectors, can be used to create a set of Lanczos vectors. The projection matrix formed by the set of Lanczos vectors transforms the system matrix A into an almost skew-symmetric, block-tridiagonal form.

First, assume that there exists a set of blocks of vectors Q_i, $i = 1, 2, \ldots, k$, that satisfy

$$A\,[Q_1\ Q_2\ \cdots\ Q_k] = [Q_1\ Q_2\ \cdots\ Q_k] \begin{bmatrix} \mathcal{F}_1 & \mathcal{G}_1 & & & \\ \mathcal{H}_1 & \mathcal{F}_2 & \mathcal{G}_2 & & \\ & \mathcal{H}_2 & \mathcal{F}_3 & \cdot & \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot \end{bmatrix} \tag{17}$$

The above equation implies the following iteration formula

$$AQ_i = Q_{i-1}\mathcal{G}_{i-1} + Q_i\mathcal{F}_i + Q_{i+1}\mathcal{H}_i \tag{18}$$

Let us further assume that the Q_i's are orthonormalized with respect to the inverse of the controllability grammian of the system. That is

$$Q_i^T W_c^{-1} Q_j = \begin{cases} I & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \tag{19}$$

Then, by premultiplying Eq. (18) by $Q_{i-1}^T W_c^{-1}$, $Q_i^T W_c^{-1}$, and $Q_{i+1}^T W_c^{-1}$ respectively, and using the orthogonality conditions in Eq. (19), it can be shown that

$$\begin{aligned} \mathcal{G}_{i-1} &= Q_{i-1}^T W_c^{-1} A Q_i \\ \mathcal{F}_i &= Q_i^T W_c^{-1} A Q_i \\ \mathcal{H}_i &= Q_{i+1}^T W_c^{-1} A Q_i \end{aligned} \tag{20}$$

Let the starting block of vectors of the iteration formula in Eq. (18) be the B matrix after being normalized with respect to W_c^{-1}. That is, let

$$Q_1 = B U_0 \Sigma_0^{-\frac{1}{2}} \quad \text{with} \quad U_0 \Sigma_0 U_0^T = B^T W_c^{-1} B \tag{21}$$

where $U_0 \Sigma_0 U_0^T$ is the singular-value decomposition of $B^T W_c^{-1} B$ with $U_0^T U_0 = I$ and Σ_0 being the diagonal matrix containing the singular values. With this special choice of starting block of vectors, the following two identities can be derived.

$$\mathcal{H}_i = -\mathcal{G}_i^T \tag{22}$$

$$\mathcal{F}_i + \mathcal{F}_i^T = \begin{cases} -\Sigma_0 & \text{if } i = 1 \\ 0 & \text{if } i \neq 1 \end{cases} \tag{23}$$

These two identities indicate that the block tridiagonal matrix in Eq. (17) is almost skew-symmetric, except the diagonal elements of $\mathcal{F}_1$.

Let

$$L \equiv \begin{bmatrix} Q_1 & Q_2 & \cdots & Q_k \end{bmatrix} \tag{24}$$

and

$$T \equiv \begin{bmatrix} \mathcal{F}_1 & \mathcal{G}_1 & & & \\ -\mathcal{G}_1^T & \mathcal{F}_2 & \mathcal{G}_2 & & \\ & -\mathcal{G}_2^T & \mathcal{F}_3 & \cdot & \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot \end{bmatrix} \tag{25}$$

Then, Eq. (17) can be written as

$$AL = LT \tag{26}$$

The orthogonality conditions in Eq. (19) imply that $L^T W_c^{-1} L = I$. Therefore, the unsymmetric system matrix A can be transformed into an almost skew-symmetric, block-tridiagonal form as shown below

$$\bar{A} \equiv L^T W_c^{-1} A L = T \tag{27}$$

The unsymmetric Lanczos algorithm proposed in this section is described by the following algorithm.

One-Sided Unsymmetric Lanczos Algorithm

Given A, B, and C.

(1) *Solve* $AW_c + W_c A^T + BB^T = 0$ *for* W_c.

(2) *Starting vectors*:

(a) $Q_0 = 0$

(b) $R_0 = B$

(c) $R_0^T W_c^{-1} R_0 = U_0 \Sigma_0 U_0^T = U_{0\alpha} \Sigma_{0\alpha} U_{0\alpha}^T$ *(SVD)*

(d) $Q_1 = R_0 U_{0\alpha} \Sigma_{0\alpha}^{-\frac{1}{2}}$ *(normalization)*

(3) *For* $i = 1,\ 2,\ \ldots,\ k-1$ *repeat:*

(e) $R_i = AQ_i - Q_{i-1}\mathcal{G}_{i-1} - Q_i\mathcal{F}_i \quad (orthog.)$

$$\mathcal{G}_{i-1} = Q_{i-1}^T W_c^{-1} AQ_i \ , \ \mathcal{F}_i = Q_i^T W_c^{-1} AQ_i$$

(f) $R_i^T W_c^{-1} R_i = U_i \Sigma_i U_i^T = U_{i\alpha} \Sigma_{i\alpha} U_{i\alpha}^T \quad (SVD)$

(g) $Q_{i+1} = R_i U_{i\alpha} \Sigma_{i\alpha}^{-\frac{1}{2}} \quad (normalization)$

end

(4) *Form the k-block transformation matrix* $L = [Q_1\ Q_2\ \ldots\ Q_k]$.

It is easy to show that the Lanczos vectors generated by the above algorithm have the following property

$$span\,[Q_1, Q_2, \ldots, Q_k] = span\left[Q_1, AQ_1, \ldots, A^{k-1}Q_1\right]$$

The sequence of vectors in the right-hand side matrix of the above equation is frequently called a set of *Krylov vectors,* which can be generated by the simple iteration formula

$$Q_{i+1} = AQ_i$$

The set of Lanczos vectors generated by the proposed algorithm is basically the orthonormalized set of the Krylov vectors. Since the starting block of vectors is chosen to be $Q_1 = BU_0\Sigma_0^{-\frac{1}{2}}$, the Lanczos vectors created are in the column subspace of $\left[\ B,\ \ AB,\ \ \cdots\ \ A^{k-1}B\ \right]$, which, in linear system terminology, is called the *controllability matrix* of the system. Therefore, the projection matrix L defined in Eq. (24) has a column subspace that is the same as that of the system's controllability matrix.

The set of Lanczos vectors generated by the Lanczos iteration formula can be used as a basis for model reduction. Let the L matrix be partitioned

into

$$L = \begin{bmatrix} L_R & L_T \end{bmatrix}$$

where $L_R \in R^{n \times r}$ corresponds to the retained portion and $L_T \in R^{n \times (n-r)}$ corresponds to the truncated portion. Then, the state vector can be decomposed into

$$z = L_R z_R + L_T z_T \tag{28}$$

The reduced system equation is described by

$$\begin{aligned} \dot{z}_R &= \bar{A}_R z_R + \bar{B}_R u \\ y &= \bar{C}_R z_R \end{aligned} \tag{29}$$

where

$$\bar{A}_R = T_R = L_R^T W_c^{-1} A L_R, \ \bar{B}_R = L_R^T W_c^{-1} B, \ \bar{C}_R = C L_R$$

Only L_R is required to produce system matrices of the reduced-order model.

Since the matrices of the reduced system satisfy $T_R + T_R^T + \bar{B}_R \bar{B}_R^T = 0$, the controllability grammian of the reduced system is an identity matrix. Also, the reduced system in Eq. (29) is a completely controllable system because the Lanczos vectors are in the controllable subspace of the system. According to the Lyapunov theorem, if a controllable system has a positive definite controllability grammian, then the system is stable. Therefore, the reduced-order model obtained by the Lanczos model reduction method is a stable system.

The advantages of the proposed one-sided, unsymmetric Lanczos algorithm over the other existing unsymmetric Lanczos algorithms are: (1) the numerical breakdown problem that usually occurs in applying the two-sided unsymmetric Lanczos method is not present, (2) the Lanczos vectors that are produced lie in the controllable and observable subspace, (3) the reduced-order

model is guaranteed to be stable, (4) a shifting scheme can be used for unstable systems, (5) the flexibility of the choice of starting vector can yield more accurate reduced-order models, and (6) the method is derived for general multi-input/multi-output systems. A linear system example and a plane truss structure example are included in Ref. [4] to show the efficacy of the proposed method.

Reference [12] describes a two-sided unsymmetric block Lanczos algorithm that generates a set of *left Lanczos vectors* and a set of *right Lanczos vectors* (analogous to sets of left eigenvectors and right eigenvectors). These two sets of Lanczos vectors form a basis that transforms the system equation to an unsymmetric block-tridiagonal form. Computational enhancements that are described in Ref. [12] produce a very robust two-sided algorithm.

2.3 Control Design Based on a Linear State Function Observer (Refs. 3, 6, 10)

A method to design low-order controllers for large-scale systems is derived from the theory of linear state function observers. The observer design problem is considered as the reconstruction of a linear function of the state vector. The linear state function to be reconstructed is the given control law. Then, based on the derivation for linear state function observers, the observer design is formulated as a parameter optimization problem. The optimization objective is to generate a matrix that is close to the given feedback gain matrix. Based on that matrix, the form of the observer and a new control law can be determined.

Consider a linear, time-invariant system of the form

$$\begin{aligned} \dot{z} &= Az + Bu, \qquad z \in R^n,\ u \in R^l \\ y &= Cz, \qquad\quad y \in R^m \end{aligned} \tag{30}$$

A linear state feedback control law has the form

$$u = Kz \tag{31}$$

where the $l \times n$ feedback gain matrix K is designed to achieve some specified performance objective by using an existing control design approach, for instance, linear quadratic control theory or the eigenvalues/eigenvectors assignment method. Due to the limited output measurement, the full state feedback control law in Eq. (31) in general cannot be realized. In order to implement the state feedback control, an observer is required to reconstruct the complete state vector. It is well known that for an n-th order system with m outputs, it is always possible to construct a reduced-order observer of order $n-m$, with poles arbitrarily placed (subject to complex paring), to yield an asymptotic estimate of the states. But, for large scale dynamic systems with very few outputs, an observer of order $n - m$ is usually too large for implementation purposes. Hence, it is desirable to design an observer with considerably reduced order r, $r < (n - m)$.

Assume that the feedback gain matrix K can be decomposed into the form

$$K = GT, \qquad G \in R^{l \times r}, \qquad T \in R^{r \times n} \tag{32}$$

where $l < r < n - m$. Then, the feedback control law in Eq. (31) becomes

$$u = GTz \tag{33}$$

which can be implemented by using the measurement of the linear state function Tz. Now, consider a r-th order observer of the form

$$\dot{q} = Eq + TBu + Fy, \qquad q \in R^r \tag{34}$$

The observer state vector q is to approximate Tz in the following asymptotic sense

$$\lim_{t\to\infty} [q(t) - Tz(t)] = 0 \tag{35}$$

It is apparent that the observer system matrices E and F cannot be arbitrary and that they must satisfy some condition(s). Define the error vector

$$\varepsilon = q - Tz \tag{36}$$

Then, from Eqs. (30) and (34),

$$\dot{\varepsilon} = Eq + FCz - TAz \tag{37}$$

If E and F are chosen to satisfy the Lyapunov equation

$$ET - TA + FC = 0 \tag{38}$$

then Eq. (37) becomes

$$\dot{\varepsilon} = E\varepsilon \tag{39}$$

The error dynamics is governed by the stability of the E matrix.

In summary, in order to realize the control law in Eq. (31) exactly, the observer described by Eq. (34) must satisfy the following three conditions:

(1) E is a stable matrix,

(2) $ET - TA + FC = 0$, and

(3) $K = GT$

If a feedback gain matrix K is already designed and given, it is reasonable to choose E and F matrices in such a way that the T matrix obtained from the Lyapunov equation (38) is "close" to K. The closeness of two matrices can be defined by the following two-norm measure

$$\gamma = \frac{\| K - GT \|_2}{\| K \|_2} \tag{40}$$

If T is close to K, then there exists a G matrix such that γ is small. The G matrix that minimizes γ is

$$G = KT^+ \tag{41}$$

where T^+ is the Moore-Penrose pseudo-inverse of T.

To determine E and F matrices so that they yield a T matrix as close to K as possible, a parameter optimization technique can be used. All of the elements of E and F are considered as optimization parameters.

Although $E \in R^{r \times r}$ and $F \in R^{r \times m}$, only $r \times (m+1)$ parameters are needed to characterize the observer system. The form of the E matrix can be chosen to be block-diagonal

$$E = diag\,(\Lambda_i) \tag{42}$$

in which

$$\Lambda_i = \begin{bmatrix} -\sigma_i & -\omega_i \\ \omega_i & -\sigma_i \end{bmatrix} \tag{43}$$

is the i-th block associated with the pair of complex conjugate poles $-\sigma_i \pm j\omega_i$. For first-order poles, the E matrix has negative real numbers on the

diagonal. With the E matrix chosen in the block-diagonal form, the Lyapunov equation (38) can be split into independent partitions as

$$\begin{bmatrix} \Lambda_1 & & \\ & \Lambda_2 & \\ & & \ddots \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ \vdots \end{bmatrix} - \begin{bmatrix} t_1 \\ t_2 \\ \vdots \end{bmatrix} A + \begin{bmatrix} f_1 \\ f_2 \\ \vdots \end{bmatrix} C = 0 \tag{44}$$

or,

$$\Lambda_i t_i - t_i A + f_i C = 0, \qquad i = 1,\ 2,\ \ldots \tag{45}$$

where t_i and f_i are the i-th partitions of T and F corresponding to the i-th block of E.

A block-by-block optimization procedure can be used to determine t_i and f_i. For the i-th partition, the optimization problem is described as:

$$\left.\begin{array}{l} \text{Minimize } \gamma(\zeta_i, \omega_i, f_i) = \dfrac{\| K - KT_i^{+}T_i \|_2}{\| K \|_2} \\ \text{where } T_i = \begin{bmatrix} T_{i-1} \\ t_i \end{bmatrix}, \text{with } t_i \text{ being the solution of} \\ \quad t_i A = \Lambda_i t_i + f_i C \\ \text{and subject to} \\ \quad \zeta_{i_{\min}} \le \zeta_i \le \zeta_{i_{\max}},\ \ \omega_{i_{\min}} \le \omega_i \le \omega_{i_{\max}} \end{array}\right\} \tag{46}$$

Starting out with $T_0 = [0]$, the observer system matrices E and F are optimized block by block with the objective of obtaining a T matrix that is close to K. Although block-by-block optimization does not yield a global minimum, it has fewer optimization parameters and the computation cost is more economic than a global optimization approach. A general criterion for choosing the bounds

for ζ_i and ω_i is to place the observer poles such that the observation process is fast enough to provide estimated states for feedback.

Based upon the above derivation, a semi-inverse observer-based control design procedure can be summarized by the following steps:

(1) Determine a state feedback gain matrix K. Set $i = 1$, and $T = [0]$.

(2) Set the bounds for ζ_i and ω_i. Solve the optimization problem (19) for t_i, Λ_i, and f_i.

(3) Expand the T matrix by adding in t_i. Check the stability of $A + BGT$ $(G = KT^+)$.

(4) Determine if the closed-loop system is stable with acceptable performance. If yes, stop; else set $i = i + 1$, and go to Step (2).

The above control design procedure is called a semi-inverse procedure because the final control law is not designed before the observer, but is a result obtained from the observer design. The observer design, however, is not completely independent of the control design, because it tries to yield a control law that is close to a given design. The dimension of the observer does not have to be specified in advance. Hence, the proposed method offers considerable flexibility for the design of a linear state feedback control law and its associated linear state function observer. As the T matrix expands in Step (3), the order of the observer grows. If, at certain point the stability property and the performance of the closed-loop system is satisfactory, the design procedure can stop there. The observer that is obtained has a dimension that is the same as

the number of row vectors in the T matrix. A four-disk system and a lightly-damped beam are used as examples in Ref. [3] to demonstrate the efficacy of the proposed control design method.

2.4 Substructure-Based Control of Flexible Structures

Earlier work on this topic is summarized in the Interim Report [Ref. 9]. Research on this topic has continued under the present grant and two companion grants (See Refs. [17] and [18]). The topic of robustness of substructural controllers was studied in Ref. [17]. Reference 18 compares the SCS method of substructural controller synthesis, which was developed under the present grant, with the CCS method described by K. D. Young in Ref. [19].

This section describes a substructure-based controller design method called *Augmented Physical Component Synthesis* (APCS). This approach evolved from an earlier substructure-based method called *Substructural Controller Synthesis* (SCS) [21]. The SCS method is very versatile because it is based on the individual substructure models and never explicitly uses the complete structure model in the controller design; this way, it can readily accomodate a changing structure. However, this same versatility can lead to a very suboptimal or unstable global controller. The APCS method uses component modal models of all of the substructures comprising the complete structure to create augmented components which, while still spatially distinct, approximate the response of a specific part of the complete structure. For example, consider a structure composed of two substructures, α, β. An augmented α component is created by appending a very low-order model of the adjacent β component. This is

depicted pictorially in Fig. 2.1.

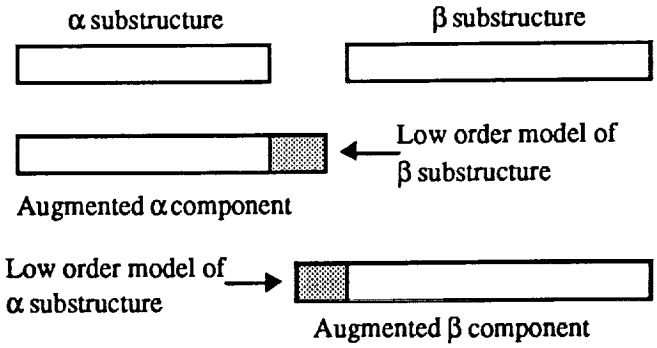


Figure 2.1: Augmented components.

An augmented β component is created similarly. When there are more than two substructures, the augmented components are created by sequentially, coupling reduced order models of the other substructures and appending them to the substructure of interest. With this method, each component will experience all of the actuators and contribute to all of the sensors. This allows for the specification of a global performance objective, which is not possible with other substructure-based methods. Essentially, the method attempts to account for the influences of a substructure's neighbors in the design of the substructure controller in order to reduce the suboptimality of the controller.

All dynamic controllers that use measurement data consist of two parts: the regulator and the observer. These are usually designed independently. First, a state feedback gain matrix is obtained for each component by some method. For example, for a linear quadratic (LQ) controller the component regulator gain matrix is obtained by minimizing a performance index associated with the augmented component, i:

$$J^i = \frac{1}{2}\int_0^\infty [z^{i^T} Q^i z^i + u^T R u]dt \tag{47}$$

Minimization of this performance index for each component produces the component's optimal feedback gain matrix, G^i. Matrix G^i can be partitioned as

$$u = -\left[G_s \;\; G_a\right]^i \begin{bmatrix} z_s \\ z_a \end{bmatrix}^i \tag{48}$$

to reflect the gains associated with the original substructure states, z_s^i, and with the appended states, z_a^i. Equation (48) manifests the contribution of the augmented component's states to all of the actuators on the complete structure. A global gain matrix will be created from the component gain matrices, so the appended states cannot participate in the feedback (i.e., $G_a = 0$) and these gains are simply truncated. The component control is then given by

$$u^i = -\left[G_s \;\; 0\right]^i \begin{bmatrix} z_s \\ z_a \end{bmatrix}^i = -\tilde{G}^i z_s^i \tag{49}$$

The states of each component contribute to every actuator, so consequently $\tilde{G}^i$ has no zero rows. Therefore, upon creating the global gain matrix, the gains associated with the appended states are replaced by the component gain matrices of the other components. The global gain matrix is created by combining the component gain matrices. For a two component example, the APCS gain matrix is obtained by

$$G^\circ = \left[\tilde{G}^\alpha \;\; \tilde{G}^\beta\right](T^T)^I \tag{50}$$

where the superscript I means a Moore-Penrose pseudoinverse. The use of $(T^T)^I$ instead of the substructure coupling matrix , T, has the effect of averaging the gains at the common interface rather that summing them. In the limiting case, when full-order models of the substructures are appended to

create augmented components, the APCS gain matrix G° is the optimal gain matrix because each augmented component is a full-order model.

The global observer is designed analogously. The state estimator of a system represented by a state space model

$$\begin{aligned} \dot{z} &= Az + Bu + \Gamma\varpi \\ y &= Cz + \upsilon \end{aligned} \tag{51}$$

is of the form

$$\begin{aligned} \dot{\hat{z}} &= A\hat{z} + Bu + L(y - C\hat{z}) \\ u &= -G\hat{z} \end{aligned} \tag{52}$$

In the APCS method, an estimator gain matrix, L^i, is designed for each component. Each matrix L^i can be partitioned according to

$$L^i = \begin{bmatrix} L_s \\ L_a \end{bmatrix}^i \tag{53}$$

to indicate the effect of the error correction term, $L(y - C\hat{z})$, on the estimates of the original substructure states and appended states. As in the regulator design, the gains associated with the appended generalized states must be removed before a global gain matrix can be formed. The global estimation gain matrix is formed by combining the component estimation gain matrices

$$L^\circ = T^I \begin{bmatrix} \tilde{L}^\alpha \\ \tilde{L}^\beta \end{bmatrix} \tag{54}$$

Again the pseudoinverse is used so that the gains at the interface states will be averaged. In the limiting case, the APCS estimator gain matrix is the same as the global optimal one.

The complete structure controller for a linear time invariant structural dynamics system is

$$\begin{aligned} \dot{\hat{z}} &= (A - BG^\circ - L^\circ C)\hat{z} + L^\circ y \\ u &= -G^\circ\hat{z} \end{aligned} \tag{55}$$

Matrices A, B, C are defined in Eq. (51) and matrices G° and L° are defined in Eqs. (50) and (54), respectively. Because no *a priori* method exisits to evaluate the closed-loop properties of the complete structure based on the closed-loop properties of substructures, stability is not guaranteed. However, since the augmented component open-loop properties approximate those of the complete structure and the component gain matrices are assembled, the quality of APCS controllers should be quite good. Results from a simulation study of a long spring-mass-damper chain are encouraging. The complete structure is created from three substructures of eleven nodes each as shown in Fig. 2.2. Some results are shown in Figs. 2.3a,b, where the performance of a global optimal LQG controller and the APCS controller are compared. The regulator objective was to minimize the position and velocity at each node. The process noise and the measurement noise intensities were set at 1×10^{-2} and 1×10^{-4}, respectively. Simulation studies for a more representative example of a large flexible structrure are currently being conducted. The results of these simulations will be published in a future CAR report.

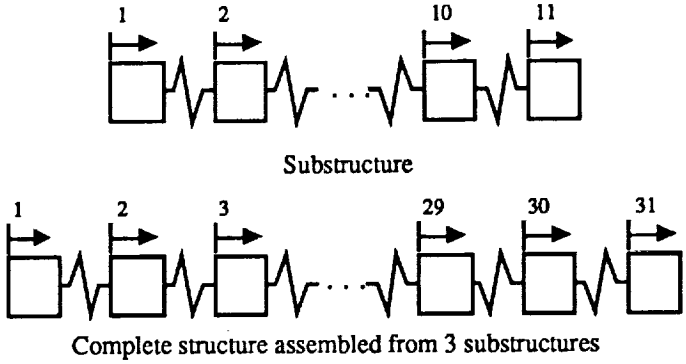


Figure 2.2: Structure used in APSC example.

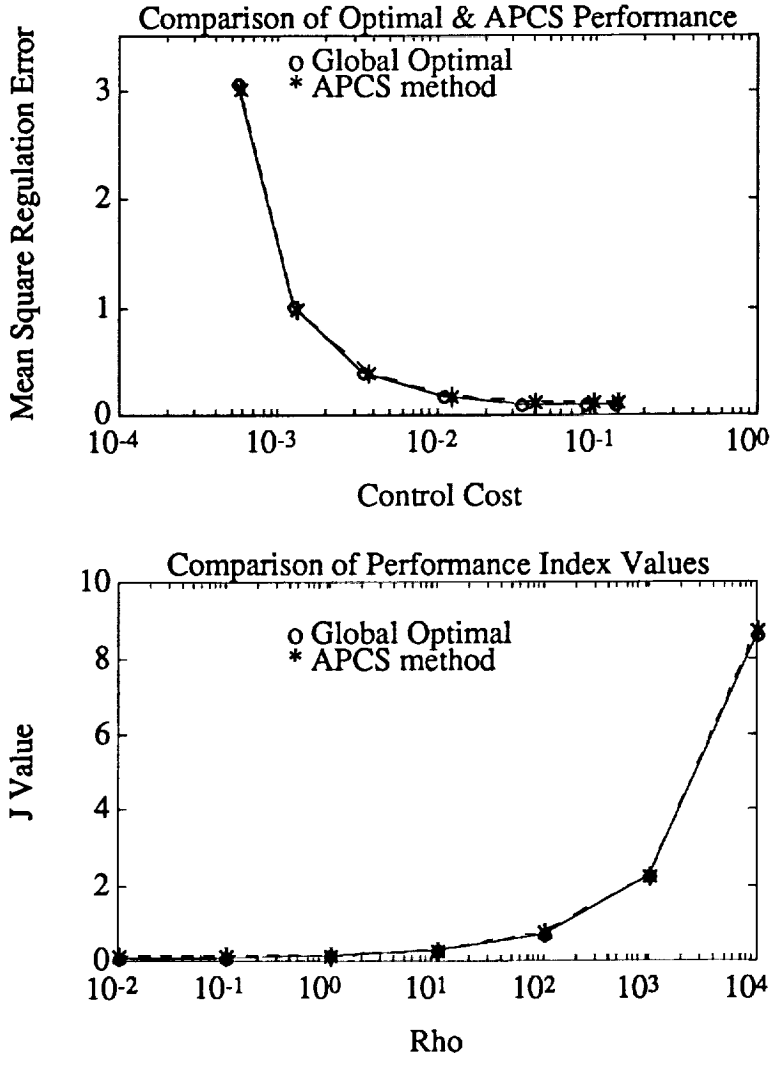


Figure 2.3: Comparison of optimal and APCS controllers.

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